

Sec. 6, is given by

$$a' = a_0 + bx, \quad (\text{A8})$$

where

$$x = \frac{1}{2}[(\cos^2\theta/\sin\theta) + \cos^2\theta/\theta].$$

Since the standard error of  $d$  and, therefore, of  $a'$  depends on the value of  $\theta$ , a weight inversely proportional to the variance of  $d$  was assigned to the  $a'$  values.

This weight was defined as

$$w_i = [1/(\sigma_{d_i}^*)^2] / \sum (1/\sigma_{d_i}^*)^2, \quad (\text{A9})$$

where

$$(\sigma_{d_i}^*)^2 = \sum_j [(d_{ij} - \bar{d}_i)^2 / (n-1)] \quad (\text{A10})$$

and  $j = 1 \cdots n_i$ , the number of  $d$ -spacing determinations for each Bragg angle  $\theta$  and  $i = 1 \cdots k$ , the number of Bragg angles (see Fig. 6).

The values of  $a_0$  and  $b$  are obtained by minimizing the sum of the squares of the deviations:

$$S = \sum_i n_i w_i (a'_i - a_0 - bx_i)^2.$$

Again, using the results from weighted least squares, we have

$$b = \sum n_i w_i a'_i (x_i - \bar{x}) / \sum n_i w_i (x_i - \bar{x})^2 \quad (\text{A11})$$

and

$$a_0 = \bar{a}' - b\bar{x}, \quad (\text{A12})$$

where

$$\bar{a}' = \sum n_i a'_i w_i / \sum n_i w_i, \quad (\text{A13})$$

$$\bar{x} = \sum n_i x_i w_i / \sum n_i w_i. \quad (\text{A14})$$

The variance of estimate is given by

$$V(a'/x) = \sum n_i w_i (a'_i - a_0 - bx_i)^2 / (n-2), \quad (\text{A15})$$

where  $b$  and  $a_0$  are given by (A11) and (A12), respectively.

The variance of  $b$  and  $a_0$  are given by

$$V(b) = V(a'/x) / \sum n_i w_i (x_i - \bar{x})^2 \quad (\text{A16})$$

$$V(a_0) = V(a'/x) [(1/n) + \bar{x}^2 / \sum n_i w_i (x_i - \bar{x})^2]. \quad (\text{A17})$$

The standard errors are then

$$\sigma_b = [V(b)]^{1/2}, \quad (\text{A18})$$

$$\sigma_{a_0} = [V(a_0)]^{1/2}. \quad (\text{A19})$$

#### APPENDIX B

##### Computation of $d$ Spacings from Incomplete Ellipses

Referring to Fig. 1 it will be seen that

$$NP = (a+b) \cot(\theta+\beta) + b \cot(\theta-\beta) \\ = a \cot(\theta+\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)] \quad (\text{B1})$$

$$NR = b \cot(\theta+\beta) + (a+b) \cot(\theta-\beta) \\ = a \cot(\theta-\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)]. \quad (\text{B2})$$

Let  $NR = l$ , for brevity, and suppose that two patterns are superimposed on the same film by making two exposures and changing the distance  $a$  from the film to the target between the exposures. Then, for example, Eq. (B2) gives

$$l_1 = a_1 \cot(\theta-\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)], \quad (\text{B3})$$

$$l_2 = a_2 \cot(\theta-\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)], \quad (\text{B4})$$

where the subscripts refer to the film position.

Subtracting (B4) from (B3),

$$l_1 - l_2 = (a_1 - a_2) \cot(\theta-\beta) = c \cot(\theta-\beta) \\ \cot(\theta-\beta) = (l_1 - l_2) / c = \Delta / c. \quad (\text{B5})$$

Here  $c$  is the distance through which the film has been shifted between exposures and  $l_1 - l_2$  is the distance between corresponding ellipses measured on the film along their common major axis.

Let us now assume that the incident radiation consists of two components of wavelength  $\lambda_1$  and  $\lambda_2$ , respectively, each giving rise to a pattern. We have then from (B5)

$$\theta_1 - \beta = \text{arccot}(\Delta_1 / c), \quad (\text{B6})$$

$$\theta_2 - \beta = \text{arccot}(\Delta_2 / c), \quad (\text{B7})$$

where the subscripts now refer to different wavelengths. Subtracting (B7) from (B6),

$$\theta_1 - \theta_2 = \text{arccot}(\Delta_1 / c) - \text{arccot}(\Delta_2 / c) \\ = \text{arccot}[(c^2 + \Delta_1 \Delta_2) / c(\Delta_1 - \Delta_2)] = \mu. \quad (\text{B8})$$

Let

$$\lambda_1 / \lambda_2 = \sin\theta_1 / \sin\theta_2 = K,$$

then

$$\sin\theta_1 = \sin(\mu + \theta_2) = K \sin\theta_2. \quad (\text{B9})$$

The second of Eq. (B9) gives finally,

$$\cot\theta_2 = (K - \cos\mu) / \sin\mu, \quad (\text{B10})$$

where  $\mu$  is given by (B8).

The last expression gives the value of the Bragg angle (and therefore of the interplanar spacing) of a given set of planes in terms of the known x-ray wavelengths, of the film shift between exposures  $c$ , and of two quantities  $\Delta_1$  and  $\Delta_2$  measured on the film (Figs. 1 and 5).

Equation (B10) may be written in a form which gives directly the  $d$  spacings, obviates the need for trigonometric tables, and is convenient for computation on a desk calculator. Using the identity  $\sin\theta = (1 + \cot^2\theta)^{-1/2}$  and the value of  $\cot\theta_2$  given by (B10),

$$d = \lambda_2 / 2 \sin\theta_2 = (\lambda_2 / 2) \\ = (\lambda_2 / 2) \\ = (\lambda_2 / 2)$$

Using the trigonometric identity  $\cot\mu = s$ , member of (B11)

$$1 + [(K/\sin\mu) - 1]$$

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A study of x-ray diffraction from thin films to obtain lognormal distribution of the crystal sizes. The investigation is carried out by the quantitative x-ray diffraction method. The results show that the distribution of the crystal sizes is lognormal and that the energy of the x-ray diffraction is proportional to the energy of the incident x-ray.

**STUDYING** microscopy and Weissmann's annealing plate-coherent with the (101) planes. Upon annealing or short annealing accumulated coherency of the film is relieved through the (101) planes. It is not possible, however, to obtain a quantitative distribution and to observe the observed

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